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# The reattachment and relaxation of a turbulent shear layer

By P. BRADSHAW AND F. Y. F. WONG
Department of Aeronautics, Imperial College, London

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Existing experiments on the low-speed flow downstream of steps and fences, and some new measurements downstream of a backward-facing step, are used to demonstrate the complicated nature of the flow in the reattachment region and its effect on the slow non-monotonic return of the shear layer to the ordinary boundary-layer state. A key feature of the flow is found to be the splitting of the shear layer at reattachment, where part of the flow is deflected upstream into the recirculating flow region to supply the entrainment; the part of the flow that continues downstream suffers a pronounced decrease in eddy length scale, evidently because the larger eddies are torn in two. This phenomenon will occur in all cases where a shear layer reattaches after a prolonged region of separation, either at low speed or in supersonic flow. For simplicity, the discussion in the present paper is confined to low-speed flows.

## 1. Introduction

Methods for calculating turbulent boundary layers and other thin shear layers are now sufficiently realistic and reliable for useful attempts to be made to calculate the more complicated flows that occur in engineering practice. Since experimenters, like theoreticians, have tended to concentrate on thin shear layers, there is a great need for data to formulate and test calculation methods. An important class of flows, and an obvious target for extension of thin-shear-layer studies, is the class of strongly perturbed shear layers: we are often interested both in the immediate response to the perturbation and in the relaxation back to the thin-shear-layer state. By definition we are usually concerned with perturbations strong enough to invalidate the boundary-layer approximation, and the most common perturbations of this sort are those involving separation and reattachment. There have been many investigations of separation 'bubbles' on aerofoils or near discontinuities in surface slope, but the few experiments on boundary-layer relaxation after reattachment all leave something to be desired; their main shortcomings are over-complicated configurations and over-optimistic interpretations of the results. The purpose of this paper is to review the existing experimental results, to present some new measurements which demonstrate the essential phenomena of reattaching flows rather more clearly than previous experiments and to comment on the causes of these phenomena, which must be represented in calculation methods.

That present-day boundary-layer calculation methods will *not* cope with flows just after reattachment was clearly demonstrated at the 1968 Stanford meeting (Kline, Morkovin, Sovran & Cockrell 1969), comparisons with the measurements of Tillman (1945) behind a square ledge ranging from poor to extremely poor. Very close to reattachment the boundary-layer approximation used in these methods is violated but the discrepancies between calculation and experiment further downstream imply that the turbulence structure takes a long time to relax to that found in an ordinary boundary layer (and in ordinary calculation models). Similar discrepancies would be expected in comparisons with the other measurements in reattaching flows.

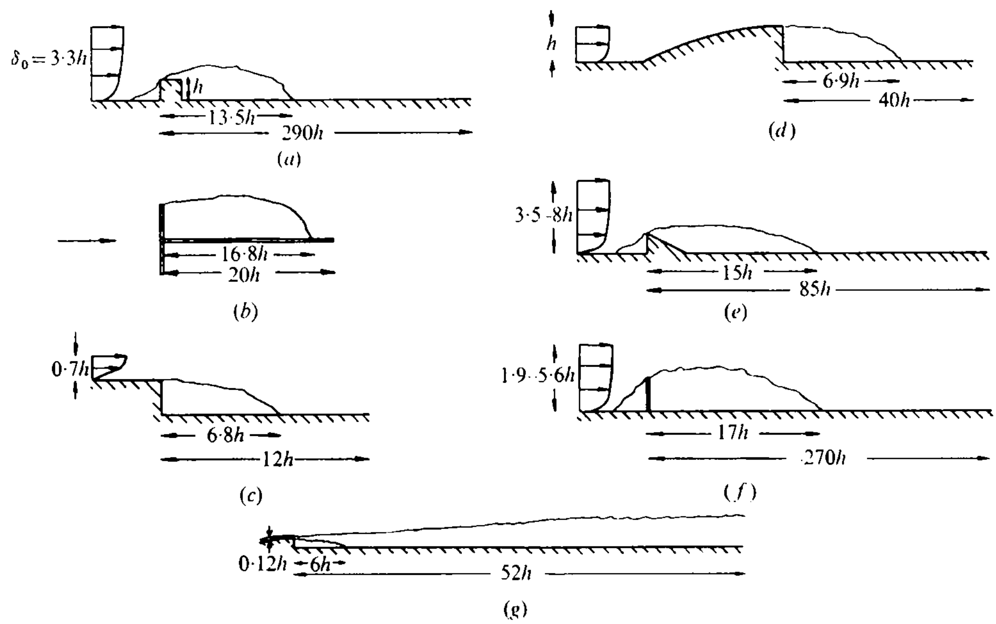


FIGURE 1. Experimental configurations. (*a*)–(*f*) not to scale. (*a*) Tillman (1945). (*b*) Arie & Rouse (1956). (*c*) Tani *et al.* (1961). (*d*) Mueller & Robertson (1963). (*e*) Plate & Lin (1964). (*f*) Petryk & Brundrett (1967). (*g*) Present experiment (to scale).

The configurations used by some previous experimenters are shown in figures 1(*a*)–(*f*). Here $\delta_0$ is the boundary-layer thickness at the obstacle position in the absence of the obstacle. The distance to reattachment and the distance to the last measurement station are shown (not to scale) as multiples of the obstacle height $h$. In every case where measurements have been made well downstream of reattachment (figures 1(*d*)–(*f*)) the initial boundary-layer thickness is of the same order as the height of the obstacle and, with the possible exception of the measurements of Mueller & Robertson (1963), *two* separations and reattachments occur. The length of the second separation region is strongly affected by the initial inclination of the dividing streamline and therefore by the details of the first separation region. For these reasons it is not possible to distinguish between the influence of the obstacle and that of the upstream boundary layer. All the recent authors including Coles (Coles & Hirst 1969), in his analysis of Tillman's (1945) data, assume that the mean velocity follows the logarithmic 'law of the wall' close to the surface, although Petryk & Brundrett (1967) note 'a small defect in the wall region' and Coles had reservations about the wall–wake formulation for Tillman's flow. All the recent authors listed in figures 1 (*b*)–(*f*) imply that the surface shear-stress coefficient returns monotonically to the constant-pressure equilibrium value and that it does so at a downstream distance no more than 50 times the height of the obstacle. The present results show clearly that neither is the case; as noted by Coles, the return to equilibrium is very slow.

The present measurements were made behind a simple backward-facing step (figure 1(*g*)) with a thin laminar boundary layer at separation, and were continued far enough downstream for the boundary layer to have returned to a nearly normal state (though not yet to an equilibrium state). Even in this, the simplest possible reattaching flow, the effects of the perturbation on the turbulence structure make conventional boundary-layer calculation methods inapplicable for many boundary-layer thicknesses downstream of reattachment. A few turbulence measurements have been made, and some indirect information about the turbulence structure has been obtained by 'numerical experiments', which alter the empirical input of a calculation method to force agreement with the experimental results. The results have been used to form a self-consistent picture of the flow; they should also be useful as a test case for future calculation methods.

Section 2 of this paper is a review of previous work, both on the relaxation region itself, and on the separated-flow and reattachment regions that constitute the perturbation. In § 3 some new measurements in the relaxation region behind a backward-facing step are presented; it appeared to us that a proper investigation of relaxation problems should start with a simple perturbation of a thin initial boundary layer. In § 4 the new data are used in a 'numerical experiment' to find the changes in the boundary-layer calculation method of Bradshaw, Ferriss & Atwell (1967) that are required to optimize agreement with the measurements (we do *not* offer the modified method for general use in relaxing flows). The resulting indirect information about the turbulence provides an important clue to the gross changes in eddy structure that occur in the reattachment region; these changes are discussed in § 5. It is concluded that the key parameter in the problem is the fraction of the shear-layer mass flow that is deflected upstream at reattachment. If this fraction is appreciable the large eddies in the shear layer are virtually torn in two, producing a turbulence structure that must differ greatly from that found in any conventional shear flow.

## 2. Review of previous experiments

### 2.1. *Definition of perturbation strength*

It is helpful to distinguish and define three strengths of perturbation that can be applied to an initially thin shear layer.

(i) A *weak perturbation* is one in which the velocity and length scales of the flow are altered without significant change in the dimensionless properties of the turbulence structure like the shapes of correlation curves or the ratio of one intensity component to another. Examples are discussed by Tani in Kline *et al.*

(1969). Generally speaking, the response to weak perturbations, such as a change of pressure gradient or roughness, should be predictable by conventional calculation methods for thin shear layers.

(ii) A *strong perturbation* is one in which the turbulence structure is significantly altered (a rough and ready measure of this being the inability of thin-shear-layer calculation methods to predict the flow) but where the flow is recognizable throughout as a perturbed form of a specific kind of shear layer. An example is a boundary layer flowing over a cavity only a few boundary-layer thicknesses long.

(iii) An *overwhelming perturbation* is one in which the shear layer changes to one of a different species, as in the mutation of a boundary layer into a wake or mixing layer.

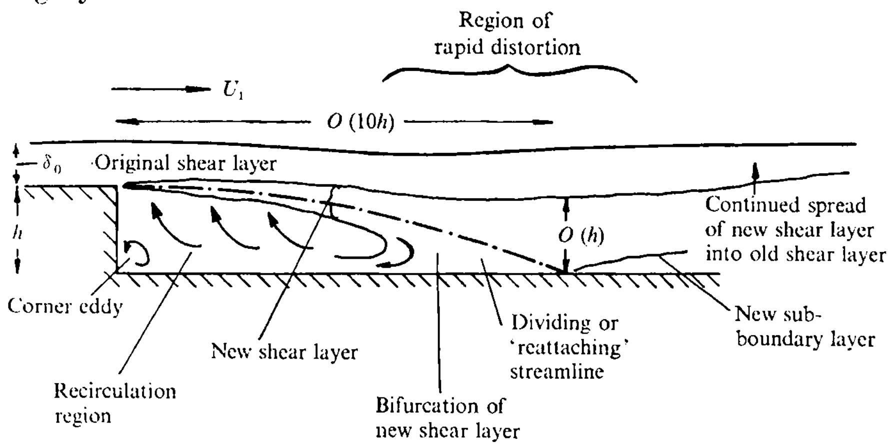


FIGURE 2. General behaviour of typical reattaching flow.

In the case of a single separation and reattachment the strength of the perturbation, as measured at the reattachment point, say, depends on how far the new shear layer that borders the reversed-flow region has spread into the original shear layer. Studies of 'internal boundary layers', the new shear layers that grow outward from the surface after a small change of roughness (weak perturbation), show that the properties on a given streamline outside the new shear layer are virtually unaffected, and unpublished measurements in a plane mixing layer by I. S. F. Jones at the National Physical Laboratory suggest that the same can be true of the flow downstream of separation. Considering for simplicity the case of flow down a backward-facing step (figure 2), in which the distance the new shear layer spreads into the original shear layer by the time reattachment occurs is very roughly equal to the step height $h$, we see that the strength of the perturbation can be classified by the value of $h/\delta_0$.

(i) Weak perturbation: $h/\delta_0 \ll 1$.

(ii) Strong perturbation: $h/\delta_0 = O(1)$.

(iii) Overwhelming perturbation: $h/\delta_0 \gg 1$.

The original shear layer is, of course, influenced by the side effects of separation, such as curvature of the mean streamlines and the region of rapid distortion induced by the pressure field near reattachment, while the new shear layer splits roughly in half at reattachment. The relative importance of all these effects is difficult to assess and it appears, paradoxically, that an overwhelming perturbation will be simpler to understand than a strong perturbation because the former is less dependent on the initial boundary layer. Strictly, the flow of a thin boundary layer over a backward-facing step involves *two* 'overwhelming' perturbations (boundary layer to mixing layer and mixing layer back to boundary layer) but the first perturbation can be ignored and the flow treated as if a fully developed mixing layer appeared at the separation point. On the other hand, existing experiments on fences or *forward*-facing steps not only have

$$h/\delta_0 = O(1)$$

but involve two separation regions (two 'strong' perturbations), and are the hardest of all to understand. The case $h/\delta_0 \ll 1$, in which the perturbation is confined to the inner layer, is discussed by Hastings (1963).

### 2.2. *The 'relaxing' flow downstream of the perturbation region*

The most useful single parameter for measuring the departure of a turbulent boundary layer from equilibrium is the Clauser parameter

$$G \equiv \left\{\int_0^\delta (U_1 - U)^2 \, dy/u_\tau^2 \delta\right\} \bigg/ \left\{\int_0^\delta (U_1 - U)\, dy/u_\tau \delta\right\} \equiv (2/c_f)^{\frac{1}{2}} (H-1)/H,$$

in the usual notation. According to the data of Coles (1962) $G$ is about 6·8 in an equilibrium constant-pressure boundary layer at high Reynolds number. In figure 3, $G$ is plotted against distance from the obstacle divided by obstacle height for the experiments of Tillman (1945), Mueller & Robertson (1963) and Petryk & Brundrett (1967). In these experiments $c_f$ was determined by assuming the validity of the logarithmic inner law, either directly (Tillman, as analysed by Coles & Hirst (1969), and Mueller & Robertson) or in the form of a Preston tube calibration (Petryk & Brundrett). Plate & Lin (1964) do not quote $c_f$ values, but only $c_f/c_{f,\,\max}$. In table 1 the distance $x_G$ between the obstacle and the point of minimum $G$ is given as a multiple of $\delta_0$, $h$ or the shear layer thickness at reattachment, $\delta_r$.

Clearly, the perturbations are neither 'weak' (scaling on $\delta_0$) nor 'overwhelming' (scaling on $h$). Indeed Tillman's and Petryk's results for distance to minimum $G$ collapse quite well on to the geometric mean, $x_G \doteq 100\sqrt{(\delta_0 h)}$ (see figure 4). Of the measurements that stop short of $x_G$, Mueller's could conceivably reach the solid line in figure 4 eventually; Plate's, which do seem to extend as far as $c_{f,\,\max}$, which is not much further upstream than $G_{\min}$, are undoubtedly different. The range of Reynolds number in these experiments was not sufficiently large to contribute much to the scatter and we must conclude that the recovery of the boundary layer does depend appreciably on obstacle shape, even when separation from a sharp edge occurs. Broadly speaking, the shape of the obstacle downstream of separation will affect the strength of recirculation in the separated-flow region. Further analysis of the data will suggest that this has a large effect on the shear layer properties downstream of reattachment. Tani, Iuchi & Komoda (1961) showed that triangular fillets (similar to Plate's 'hill' but shallower), and obstructions in the recirculation region, appreciably changed the pressure

| | $x_r/h$ | $\delta_0/h$ | $\delta_r/h$ | $x_G/h$ | $G_{min}$ | $x_G/\delta_0$ | $x_G/x_r$ | $x_G/(\delta_0 h)^{\frac{1}{2}}$ |
|---|---|---|---|---|---|---|---|---|
| Tillman | 13·5 | 3·3 | 4·2 | 160 | 4·784 | 49 | 12 | 88 |
| Mueller & Robertson, $\frac{1}{4}$ in. | 7 | 1·0 | 2·4? | > 40 | < 5·3 | > 40 | > 6 | > 40 |
| Petryk & Brundrett, $\frac{1}{4}$ in. | 17 | 5·6 | 5·8 | 230 | 5·56 | 41 | 13·5 | 97 |
| Petryk & Brundrett, $\frac{3}{8}$ in. | 17 | 3·74 | 4·7 | 200 | 5·24 | 53·5 | 12 | 104 |
| Petryk & Brundrett, $\frac{1}{2}$ in. | 17 | 2·8 | 3·9 | $\geqslant$ 180 | $\leqslant$ 5 | $\geqslant$ 65 | $\geqslant$ 10·6 | 108 |
| Petryk & Brundrett, $\frac{5}{8}$ in. | 17 | 2·24 | 3·5 | $\geqslant$ 150 | $\leqslant$ 4·7 | $\geqslant$ 67 | $\geqslant$ 8·8 | 100 |
| Petryk & Brundrett, $\frac{3}{4}$ in. | 17 | 1·87 | 3·4 | 140 | 4·54 | 75 | 8·2 | 102 |
| Plate & Lin, figure 15 | 22 | 8 | 8 | > 80 | — | > 10 | > 4 | — |
| Plate & Lin, figure 17 | 15 | 3·5 | 4 | > 80 | — | > 23 | > 5 | — |
| Arie & Rouse | 16·8 | 0 | 4·5 | Measurements near reattachment only | | | | |
| Tani *et al.* | 7 | 0·7 | 1·5 | | | | | |
| Wong | 6 | 0·12 | 1·5 | 35 | 5·5 | 320 | 60 | 100 |

TABLE 1. Details of previous experiments. $x_r$ = distance to reattachment, $\delta_0$ = boundary-layer thickness upstream of obstacle, $\delta_r$ = thickness at reattachment, $x_G$ = distance to minimum $G$

distribution near reattachment. These results imply that attempts to correlate properties of the relaxation regions downstream of different obstacles in terms of a few parameters describing the perturbation are not likely to succeed, despite the success of Good & Joubert (1968) in correlating properties of the flow *near* a simple fence.

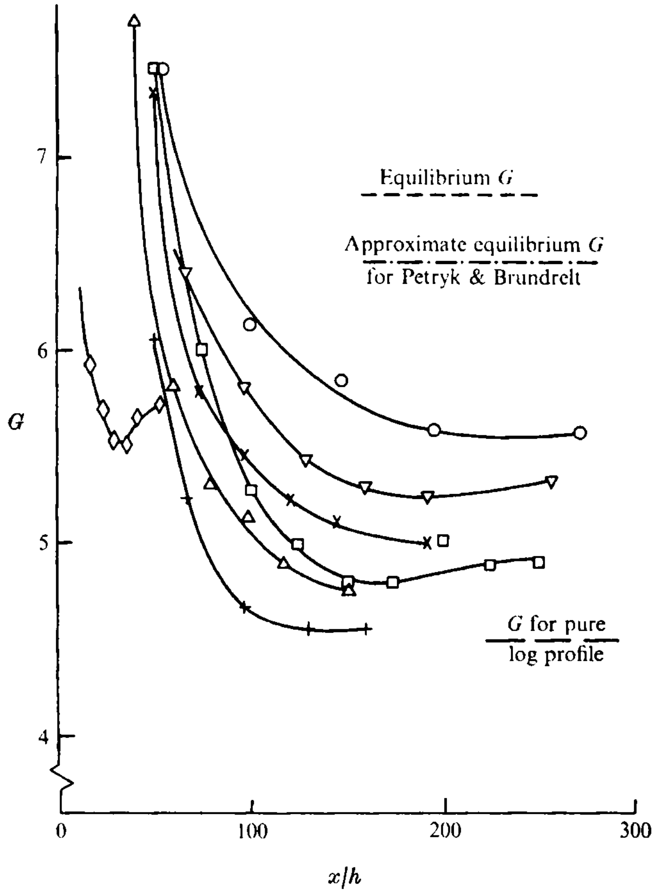


FIGURE 3. Clauser parameter $G$ versus distance from obstacle. □, Tillman. Petryk & Brundrett: ○, $\delta_0/h = 5{\cdot}6$; ▽, 3·74; ×, 2·8; △, 2·24; +, 1·87. ◇, present results.

Figure 3 shows that $G$ (or $c_f$) does *not* return monotonically to its equilibrium value (which would be about 6·5 in the slightly accelerated flow of Petryk & Brundrett). It is surprising that Mueller & Robertson, Plate & Lin and Petryk & Brundrett should *all* have claimed a monotonic return to equilibrium, although the first two authors had an insufficiently long test section to observe the increase in $G$ at large $x$.

The velocity profiles presented by Tillman, Plate & Lin and Petryk & Brundrett can, with hindsight and the measurements to be described below, be seen to disobey the logarithmic inner law, not only in the region close to reattachment, where even the most guileless would expect it to fail, but for many boundary-layer thicknesses downstream. Coles (Coles & Hirst 1969) fitted a logarithmic law to Tillman's data and implied by omission that he regarded the fit as acceptable in view of the experimental difficulties. It is just possible that the discrepancies (figure 5) are caused by the influence of high turbulence on the Pitot

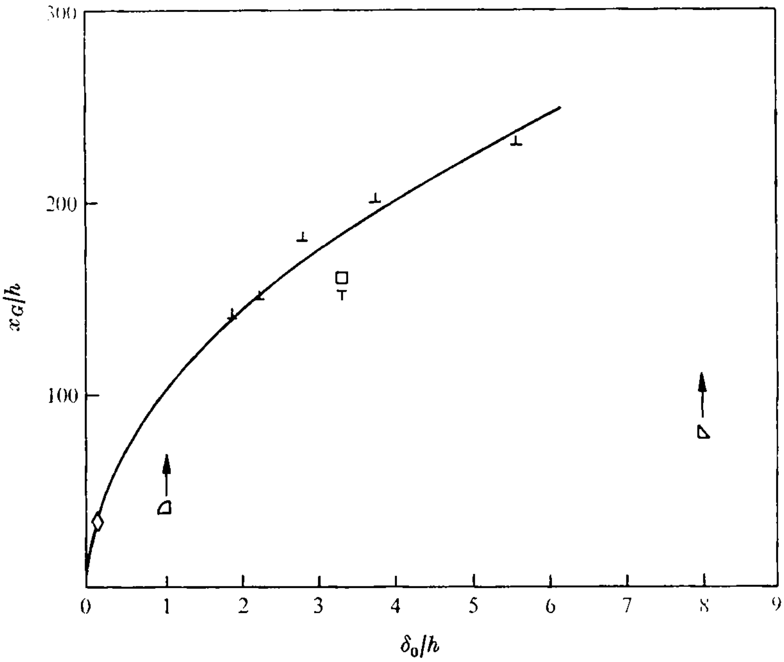


FIGURE 4. Distance $x_G$ to minimum $G$ versus initial boundary-layer thickness. □, Tillman; ◿, Mueller & Robertson; △, Plate & Lin; ⊥, Petryk & Brundrett; ◇, present results; —, $x_G = 100(\delta_0 h)^{\frac{1}{2}}$.

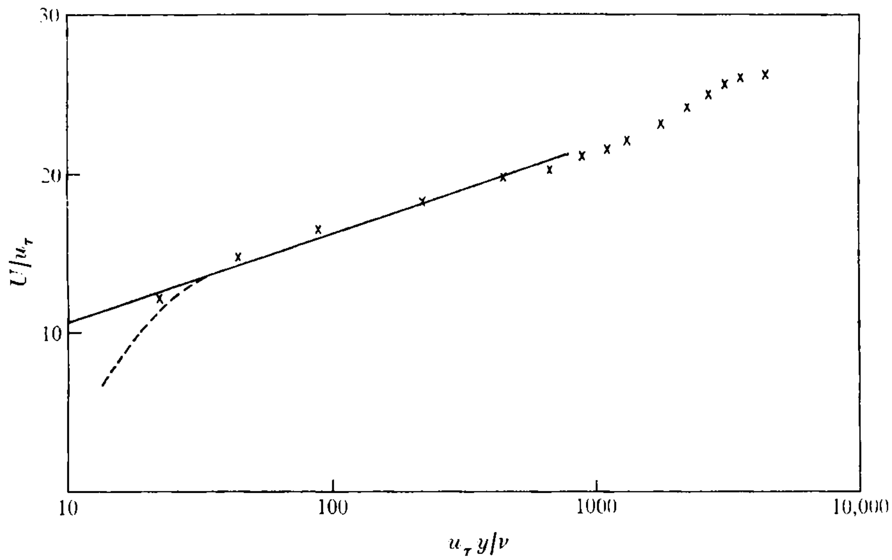


FIGURE 5. Semi-logarithmic plot of Tillman's profile at $x/h \approx 60$ (from Coles & Hirst (1969), cf. present results at $x/h = 52$, figure 8(d)).

tubes, but the discrepancies show the wrong trend with $y$ and are much larger than if the tube had read high by a quantity of order $\frac{1}{2}\rho(\overline{u^2}+\overline{v^2}+\overline{w^2})$, as is normally assumed. In any case, if the turbulence is high enough to cause such gross measuring errors one would not trust the local-equilibrium assumptions which provide the most satisfying justification for the logarithmic law.

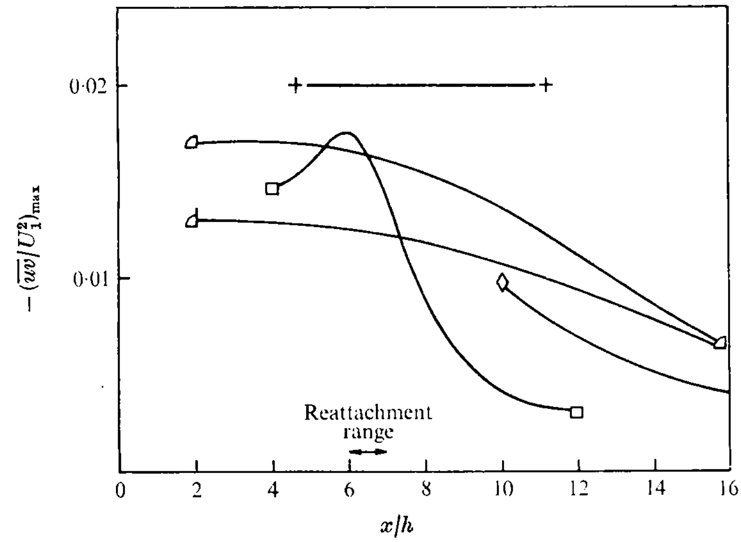


FIGURE 6. Maximum shear stress in shear layer. +, Arie & Rouse; □, Tani *et al.* Mueller & Robertson: ◿, $\delta_0/h$ = 0·5; ◿, $\delta_0/h$ = 0·33. ◇, present results (*measurement* at $x/h$ = 10, *calculation* thereafter).

The turbulence measurements of Mueller & Robertson and of Plate & Lin in the relaxation region are qualitatively what might be expected; we do not know enough about turbulence to identify any unusual features analogous to departures from the logarithmic velocity profile. The behaviour of the maximum shear stress *near* reattachment, as measured by Tani *et al.* (1961) and Mueller & Robertson (figure 6) is surprising in two respects. First, the shear stress in the free shear layer is significantly higher than the value of about $0{\cdot}01\,\rho U_1^2$ found in a plane mixing layer, despite the fact that the streamline curvature is in a stabilizing sense, and there are quite large differences between different cases. The boundary layers of Mueller & Robertson and of Tani *et al.* were turbulent at separation, while Arie & Rouse (1956), with the highest shear stress of all $(0{\cdot}02\rho U_1^2)$, had a very thin laminar boundary layer at separation. Part of the reason for the large values of shear stress is that the effective velocity difference across the free shear layer is *not* the free-stream velocity $U_1$; the excess velocity outside the separated flow region is small (except perhaps in Arie & Rouse's case) but there is always a significant backflow in the separated region, which is *not* a 'dead-water' region (see Tani *et al.* 1961, figure 10). However, the reversed-flow velocity does not seem to exceed 0·2 of the external-flow velocity but the shear stress *does* seem to exceed $(1+0{\cdot}2)^2$ times the plane shear layer value, so that this is not a complete explanation. The measurements of Arie & Rouse, and of Mueller & Robertson, show that the shear stress in the reversed-flow region is not negligible, so possibly the shear stress in the shear layer is augmented by 'feedback', that is, the re-entrainment of stress-bearing fluid. However the details of such a mechanism are obscure.

The second curious feature of the shear stress near reattachment is the rapid decrease of the maximum shear stress within the shear layer (figure 6). The mean velocity gradient $\partial U/\partial y$ on a given streamline will be nearly the same before and after the region of rapid distortion near reattachment (mean vorticity is nearly conserved), so that one would at first expect the velocity gradient and shear stress in the outer part of the reattached shear layer to remain relatively constant until affected by the internal boundary layer growing from the surface. However it seemed most unlikely that both sets of measurements shown in figure 6 are grossly in error, and it was found necessary to simulate the same rapid decrease in $-\overline{uv}_{max}$ to obtain the best agreement between a calculation method and the present measurements (see figure 6 and § 4). These results are strong evidence that large changes in turbulence structure occur when the shear layer bifurcates at reattachment.

Further discussion of the turbulence structure of the reattachment and relaxation regions will be deferred until after a description of the present experimental results, which have the advantage of a simple configuration, a long test section and a set of fairly reliable surface shear-stress measurements.

## 3. The present measurements in the relaxation region

### 3.1. *Apparatus*

The measurements were made in an open-circuit wind tunnel driven by a centrifugal blower at entry. The turbulence level is about 0·07 per cent. The working section is 75 cm wide, 290 cm long and, nominally, 12·5 cm high; the actual height of the working section can be adjusted to obtain the desired pressure gradient by means of hand-operated jacking screws attached to the flexible roof. The backward-facing step, of height 2·5 cm, was formed by a fairing in the contraction, so that the actual height of the contraction exit was 10 cm. The roof height was adjusted to give zero pressure gradient on the floor downstream of the reattachment region but no attempt was made to simulate an infinite stream in the region near the step.

The measurements were made at a tunnel speed of 25 ms$^{-1}$ in the relaxation region. The boundary layer at the edge of the step was laminar and 0·32 cm thick. A plane mixing layer takes a distance of roughly 100 times its initial thickness to attain self-preservation after laminar separation (Bradshaw 1966); in the present case this distance is 30 cm whereas the distance to reattachment is only about 15 cm, so that even with the thinnest possible laminar boundary layer there is some effect on the mixing layer at reattachment.

Mean velocity profiles were measured with a round Pitot tube of 0·114 cm O.D., and Preston tubes of diameters 0·091, 0·122, 0·204 and 0·287 cm were used for surface shear-stress measurements. Further details of the tunnel and apparatus are given by Wong (1970). A few turbulence measurements were made, using a

Disa 55A38 cross-wire probe and 55D01 constant-temperature anemometers (without linearizers), and intermittency was deduced by computer processing of digital records (Antonia & Bradshaw 1971).

### 3.2. *Results*

Measurements of surface shear stress, using Preston tubes of various sizes with Patel's (1965) calibration, are shown in figure 7. The $c_f$ indicated depends slightly on tube diameter but the differences between the largest and smallest tubes are generally less than three per cent. The mean line chosen favours the smaller tubes, which are less likely to be affected by departures of the inner-layer behaviour from that of a conventional boundary layer.

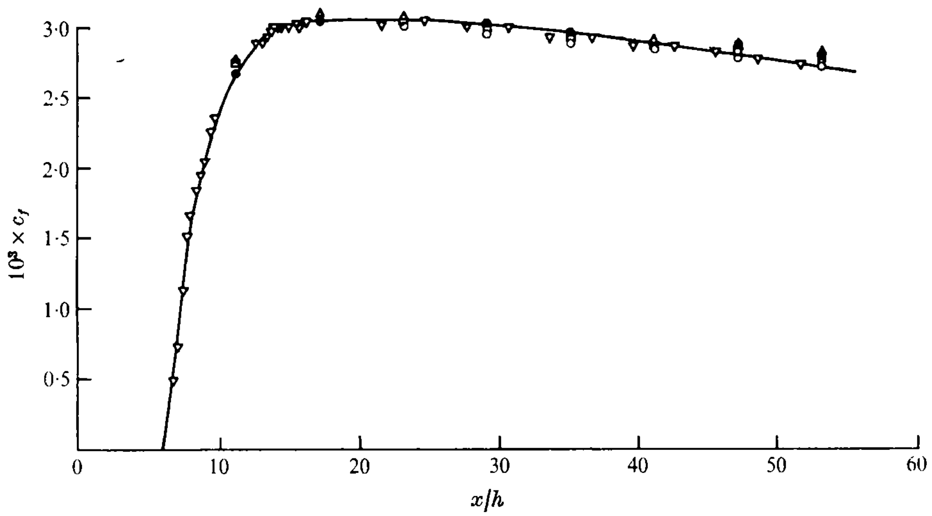


FIGURE 7. Local skin friction coefficient in present experiment. Preston tube diameter: ○, 0·091 cm; ▽, 0·109 cm; □, 0·122 cm; △, 0·204 cm; ●, 0·287 cm.

That such departures *do* occur is clear from the mean velocity profiles shown in figure 8 (cf. figure 5). The profiles show a marked dip below the universal 'inner law' profile and it is clear that they would not coincide with it throughout the inner layer whatever the choice of $c_f$. The coincidence near the surface is of course forced by using a $c_f$ derived from Preston tubes (except at $x/h = 10$, near reattachment, where obvious errors have occurred), but economy of hypothesis suggests that the profiles are likely to depart monotonically from the inner law as $y$ increases and the measurements certainly show a basically monotonic *return* to it as $x$ increases. However, even at the last station, 52 step heights downstream of the step, the profile is still very different from that in a conventional boundary layer. The comparatively close agreement (table 2) between the measured $c_f$ and values derived from the profile family of Thompson (1965) and the Ludwieg–Tillman formula

$$c_f = 0{\cdot}256 \times 10^{-0{\cdot}678H} \times R_\theta^{-0{\cdot}268}$$

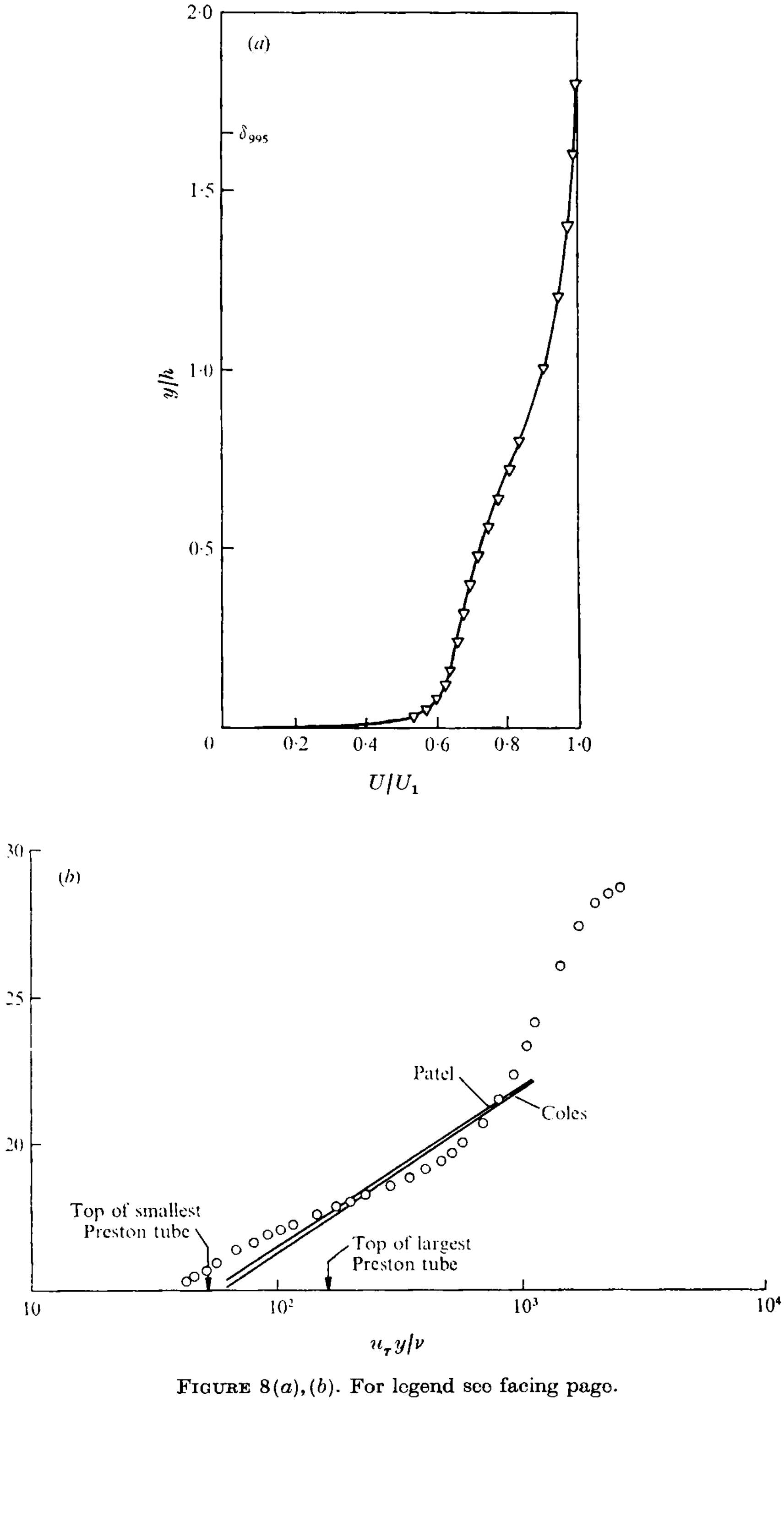


FIGURE 8(a), (b). For legend see facing page.

is surprising at first sight (the missing entries correspond to $R_\theta$ and $H$ outside Thompson's 'limiting curve' marking the range of validity of his formula). In fact rather spectacular profile changes would be needed to upset the relation between the surface shear stress and the integral parameters, providing that the surface shear stress and the velocity fairly close to the surface are still connected by the logarithmic law. Mueller & Robertson show that the same applies to the relation between *two* integral parameters.

The dip in the velocity profile is even more surprising when one remembers that, at least near reattachment, the shear-stress gradient $\partial\tau/\partial y$ will be positive

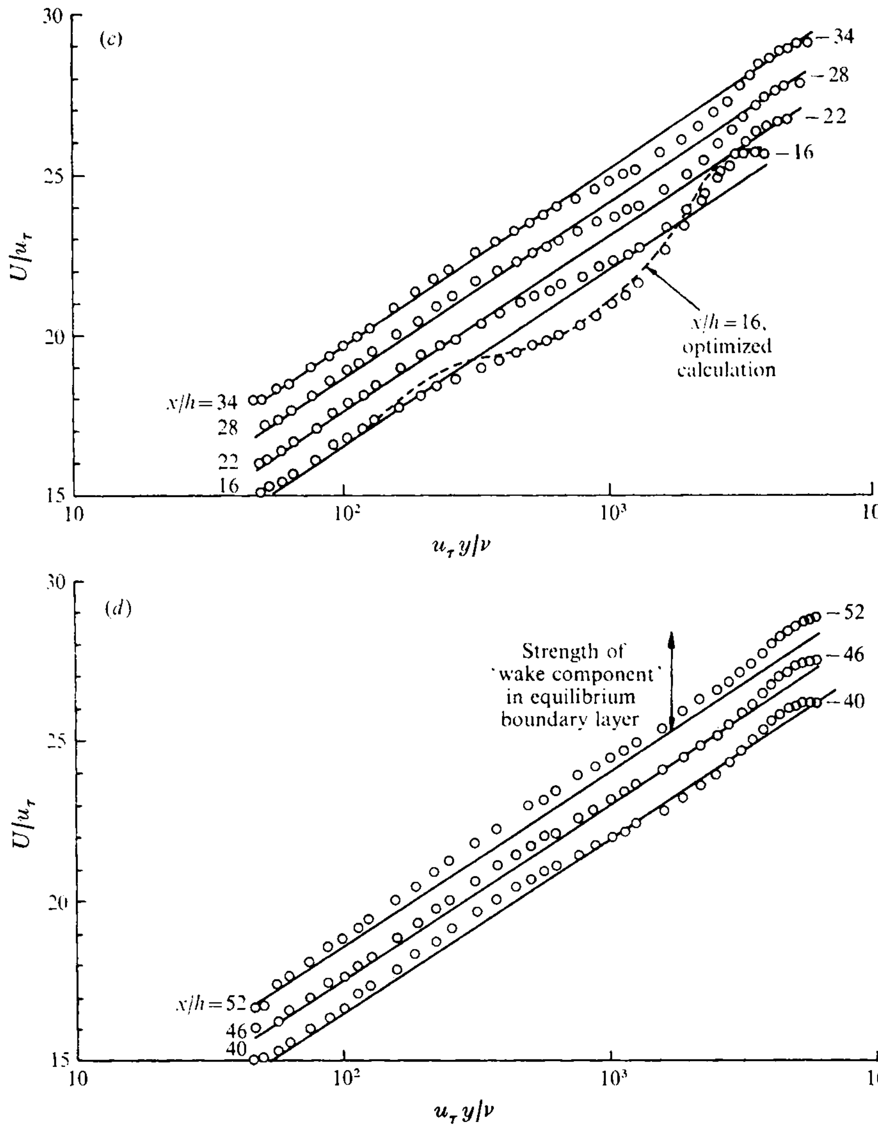


FIGURE 8. Mean velocity profiles. (*a*) $x/h$ = 10, linear plot, (*b*) $x/h$ = 10, semi-logarithmic plot, (*c*) $x/h$ = 16–34, semi-logarithmic plot. Scale of $U/u_\tau$ refers to $x/h$ = 16; subsequent curves displaced by one unit each time. (*d*) $x/h$ = 40–52.

| $x$, cm | 25 | 40 | 55 | 70 | 85 | 100 | 115 | 130 |
|---|---|---|---|---|---|---|---|---|
| $x/h$ | 10 | 16 | 22 | 28 | 34 | 40 | 46 | 52 |
| $U_1$, cm $s^{-1}$ | 2449 | 2465 | 2459 | 2434 | 2452 | 2445 | 2465 | 2457 |
| $\delta_{995}$, cm | 4·3 | 5·2 | 6·2 | 7·0 | 7·65 | 8·1 | 8·5 | 8·6 |
| $\delta^*$, cm | 0·73 | 0·714 | 0·745 | 0·781 | 0·838 | 0·869 | 0·90 | 0·915 |
| $\theta$, cm | 0·512 | 0·549 | 0·580 | 0·614 | 0·661 | 0·682 | 0·707 | 0·720 |
| $R_\theta \times 10^{-4}$ | 0·845 | 0·911 | 0·961 | 1·005 | 1·092 | 1·123 | 1·174 | 1·190 |
| $H$ | 1·424 | 1·30 | 1·285 | 1·272 | 1·268 | 1·274 | 1·273 | 1·270 |
| $G$ | 8·55 | 5·93 | 5·70 | 5·53 | 5·51 | 5·65 | 5·67 | 5·72 |
| $c_f \times 10^3$ | 2·43 | 3·04 | 3·04 | 3·00 | 2·95 | 2·90 | 2·84 | 2·77 |
| $\theta_{10} + \int_{10}^{x} \frac{1}{2} c_f \, dx$ | 0·512 | 0·543 | 0·566 | 0·589 | 0·609 | 0·628 | 0·647 | 0·664 |
| $(2/c_f)^{\frac{1}{2}}$ | 28·69 | 25·65 | 25·65 | 25·82 | 26·04 | 26·26 | 26·54 | 26·87 |
| $u_\tau/\nu$, $cm^{-1}$ | 575·0 | 647·2 | 645·7 | 634·7 | 634·1 | 627·0 | 625·4 | 615·9 |
| $c_f \times 10^3$ (Ludwieg-Tillman) | 2·361 | 2·807 | 2·833 | 2·857 | 2·811 | 2·764 | 2·736 | 2·739 |
| $c_f \times 10^3$ (Thompson) | 2·25 | 2·90 | 3·00 | > 3·00 | — | — | — | 3·00 |

TABLE 2. Integral parameters and other data for present experiment. $U_1$ is the velocity outside the shear layer, $\delta^*$ and $\theta$ are the displacement and momentum thicknesses

near the surface as the shear stress rises from a surface value typical of a boundary layer to a maximum value more typical of a free shear layer. (Note that these measurements, shown in figure 9, were made about 4 step heights downstream of reattachment.) This, according to the local-equilibrium form of the mixing length formula (Townsend 1961),

$$\frac{\partial U}{\partial y} = \frac{(\tau/\rho)^{\frac{1}{2}}}{Ky},$$

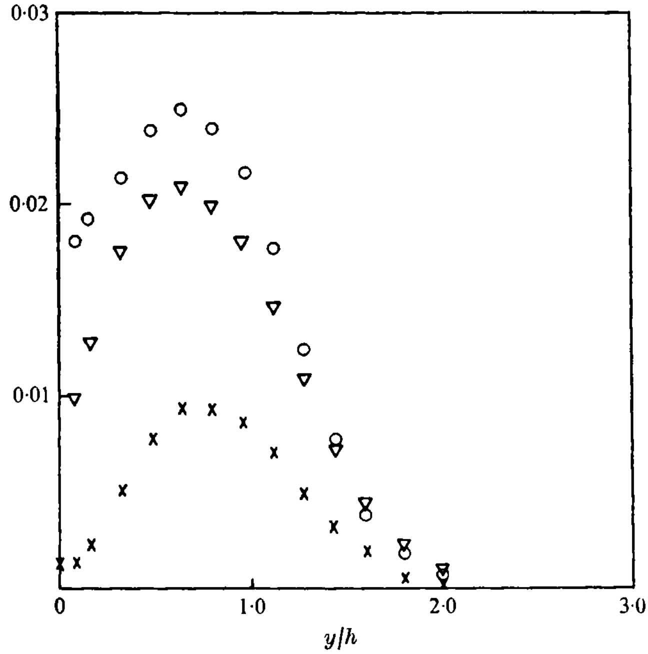


FIGURE 9. Turbulence measurements at $x/h = 10$. ○, $\overline{u^2}/U_1^2$; ▽, $\overline{v^2}/U_1^2$; ×, $-\overline{uv}/U_1^2$.

would give a *larger* velocity gradient than the 'logarithmic' value $(\tau_w/\rho)^{\frac{1}{2}}/Ky$, whereas the experimental value is of course smaller in the region $y < 0{\cdot}2\delta$, where the above formula should hold. In view of the almost universal use of this formula, or the 'log law', in calculation methods, it is important to examine possible reasons for the failure of the local-equilibrium formula. The main possibilities are (i) that the turbulence is not in local (energy) equilibrium but is changing rapidly in the streamwise direction and (ii) that the length scale of the turbulence is not proportional to $y$. Now the boundary-layer calculation method of Bradshaw, Ferriss & Atwell (1967) can be regarded as an improvement on the mixing-length formula by allowance for departures from local equilibrium, still assuming that the turbulence length scale is $Ky$ in the inner layer. The fact that predictions by this method do *not* show a dip in the velocity profile indicates that the actual length scale is not $Ky$, but increases much more rapidly with $y$. This is plausible because at reattachment the length scale, being typical of the mixing layer, is presumably constant down to a small value of $y$.

The thickness of the region in which the mean velocity *does* follow the logarithmic profile – presumably a local-equilibrium region with length scale proportional to $y$ – is roughly $0{\cdot}008x$ and approaches the value $0{\cdot}1\delta$, typical of an ordinary boundary layer, at about $x/h = 50$. This growth rate of the logarithmic region is very nearly the same as that downstream of a small change of surface roughness (see Peterson (1969) for a good general review). Since the latter is a much smaller perturbation than separation and reattachment, one is tempted to suppose that the rate at which a local-equilibrium region re-establishes itself is almost independent of the turbulent flow that provides its outer boundary condition. The only obviously necessary condition for this is the usual necessary condition for the existence of a local-equilibrium region, that its typical vorticity fluctuation (ratio of root-mean-square intensity to integral length scale, say) shall be much larger than the typical vorticity fluctuation of the turbulence beyond the outer boundary. However it remains to be seen whether this useful extension of the local-equilibrium concept can be substantiated.

The variation of the Clauser parameter $G$ with downstream distance, shown in figure 3, is roughly the same as in the fence experiments, but the minimum value is higher and the recovery quicker in terms of $x/h$. Evidently the 'overwhelming perturbation' caused by a step with $h/\delta_0 \gg 1$ causes less severe disturbance to the relaxing boundary layer than does a 'strong perturbation' with $h/\delta_0 = O(1)$, though, of course, recovery from the latter is quicker for a given boundary-layer thickness $\delta_0$ upstream of the perturbation.

One cannot deduce much about the outer-layer turbulence from mean velocity profiles alone, but since the lifetime of the outer-layer eddies is longer than that of the small eddies in the inner layer we may expect the outer layer to take even longer than the inner layer to relax to the ordinary boundary-layer structure. Positive evidence for this is provided by measurements of the intermittency at $x/h = 30$ shown in figure 10($a$), where they are compared with the measurements of Klebanoff (1955) in a constant-pressure boundary layer. It is difficult to compare the results with the intermittency in a self-preserving mixing layer. The fairest basis seems to be to plot intermittency against $U/U_1$ but figure 10($b$) shows large differences between mixing layer, boundary layer and step flow; if anything, the intermittency in step flow is nearer to that in a mixing layer, even at $x/h = 30$. The fact that the intermittency lies *between* that of the boundary layer and that of the mixing layer indicates that no disturbances of abnormally large scale ('unsteadiness') arise in the reattachment region.

It is interesting to compare the behaviour of the flow after reattachment with that in a boundary layer relaxing from a strong adverse pressure gradient. In the experiment of Bradshaw & Ferriss (1965; see run 2400 of Coles & Hirst 1969) a retarded boundary layer with a maximum shear stress of $0{\cdot}0033\rho U_1^2$ and a surface shear stress of $0{\cdot}0006\rho U_1^2$ (cf. figure 9) entered a region of constant pressure. The surface shear stress rose monotonically and had nearly reached the equilibrium value of about $0{\cdot}001\rho U_1^2$ at 35 boundary-layer thicknesses downstream of the end of retardation. The ratio of maximum shear stress to constant-pressure wall stress is not an order of magnitude less than that in the reattaching flows but the relaxation process is much less spectacular (Coles (Coles & Hirst

1969) suggests that the logarithmic law breaks down in this flow but the original authors showed that turbulence effects on *flattened* Pitot tubes were responsible). It seems that much of the difference is caused by the rapid distortion of the step flow in the reattachment region itself.

The *qualitative* features of the reattached boundary layer are as follows.

(i) A local-equilibrium layer, following the logarithmic law, which spreads out

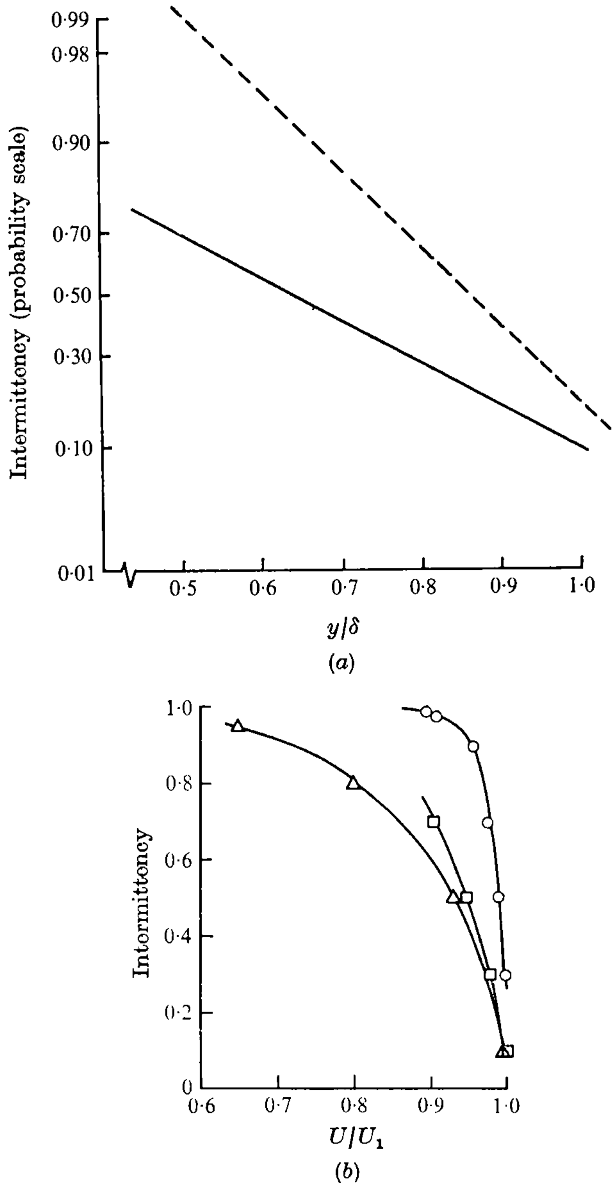


FIGURE 10. Intermittency. (*a*) Intermittency versus $y/\delta$. —, present results, $x/h = 30$; – – –, equilibrium constant-pressure boundary layer (Klebanoff 1955). (*b*) Intermittency versus $U/U_1$. □, present results, $x/h = 30$; ○, boundary layer (Klebanoff 1955); △, mixing layer (Antonia & Bradshaw 1971).

from the surface at about the same rate as after a *weak* perturbation caused by a change of roughness.

(ii) A layer in which the apparent mixing length, or a true length scale of the turbulence like the dissipation length parameter, increases rapidly above the local-equilibrium value with increasing $y$. This near-discontinuity in length scale evidently arises from the bifurcation of the mixing layer at reattachment which brings what was previously the central region of the mixing layer into close contact with the surface.

(iii) An outer layer which, except for the effects of the rapid distortion near reattachment, will retain the characteristics of the mixing layer until the effects of the altered boundary condition at the surface propagate through it.

## 4. Optimization of an existing calculation method

To obtain some quantitative information about the differences in turbulence structure between reattached shear layer and an ordinary boundary layer we have examined the modifications to the dissipation length parameter $L \equiv (-\overline{uv})^{\frac{3}{2}}/$(dissipation rate), the most important of the empirical turbulence functions used by Bradshaw, Ferriss & Atwell (1967), required to improve agreement between calculations and the present experiment. This may seem a poor alternative to turbulence measurements, but in fact it gives us information about a turbulence length scale $L$ which would be difficult to get in any other way. It may be mentioned that Bradshaw, Ferriss & Atwell explicitly stated that the (unmodified) empirical functions "would not in general be valid in...boundary layers just downstream of reattachment".

The calculations were started at $x/h = 10$, about two boundary-layer thicknesses downstream of reattachment. The calculations include an allowance, in the continuity equation, for the effects of lateral convergence in the plane of the boundary layer, and differ therein from the results given by Wong (1970); the virtual origin of the flow was about 10 metres ($400h$) downstream so that the convergence was too small to account for any major changes in the turbulence structure. The optimization of $L$ was carried out using a search program written by Dr G. D. Huffman, to whom we are indebted for helpful advice. $L$ was specified by a piecewise-analytic formula as shown in figure 11. The actual values chosen for the coefficients were functions of $x$:

$$y_b/\delta = 0{\cdot}01(x-x_r)/\delta,$$

$$\frac{d\Delta L}{dy} = 2\delta/(x-x_r),$$

$$L_{\max}/\delta = 0{\cdot}095(1-0{\cdot}3\exp\{-(x-x_r)/40h\}),$$

$x_r$ being $6h$ and $\delta$ being of the order of 2–$3h$. The 'time constant' of $40h$ is equal to about 25 times the boundary-layer thickness at reattachment. Figure 12 shows the results for the original $L$, the optimum $L$ and a run in which the 'time constant' of $L_{\max}$ was changed from $40h$ to $80h$, which made surprisingly little difference. The velocity profiles (figure 8, $x/h = 16$) exhibited a rather more abrupt version of the dip below the logarithmic law found in the experiments. This could have been improved by choosing a smoother distribution of $L$, but our object was to assess the gross changes in turbulence structure rather than to

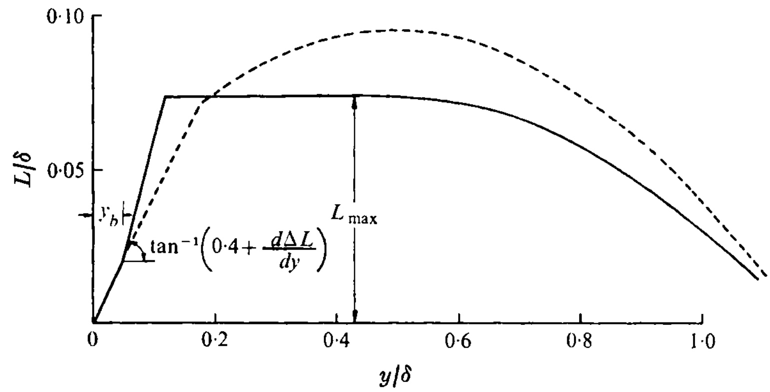


FIGURE 11. Dissipation-length parameter $L$. 'Standard' distribution compared with optimum distribution at $x/h$ = 16. —, 'modified' $L$; - - -, 'standard' $L$.

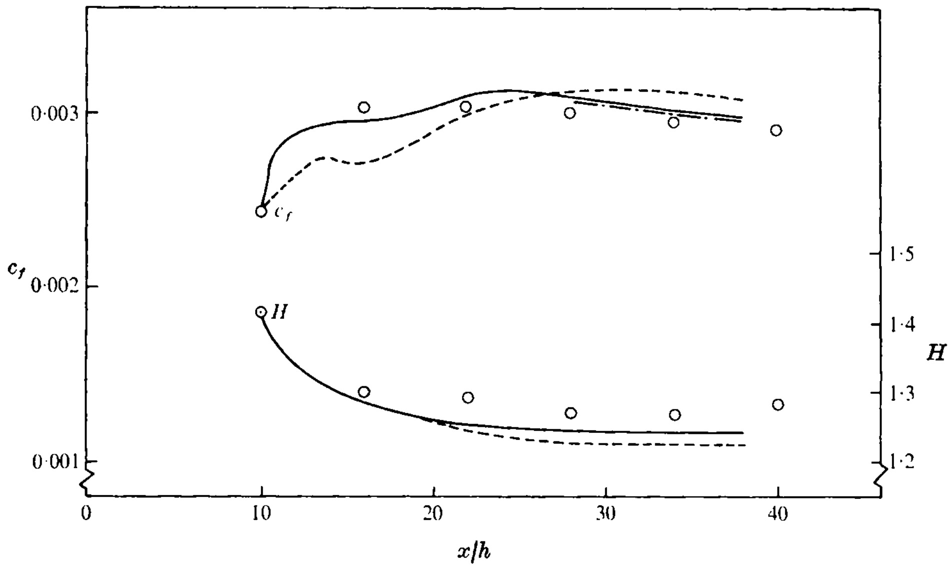


FIGURE 12. Results of optimized calculations. ○, experiment; - - -, 'standard' $L$; —, optimum $L$; —·—·—, 'optimum' $L$ with time constant doubled.

develop a usable calculation method. An illustration of the dangers of trying to extend the validity of a calculation method in this way is provided by the failure of the above modifications to effect much improvement in the comparisons with Tillman's measurements, where even larger changes in turbulence structure occur.

## 5. The reattachment region

A good deal of further work would be needed to document and explain all the features of the reattachment region (figure 2); this is a discussion of those features that affect the reattached boundary layer and is based partly on observations of the latter.

### 5.1. *The free shear layer*

The differences between the separated shear layer and the plane mixing layer (between a uniform stream and still air) seem to be large even when the influence of the initial boundary layer is negligible.

(i) Streamline curvature in the $x, y$ plane tends to *reduce* the shear stress and turbulence intensity (Wyngaard, Tennekes, Lumley & Margolis 1968).

(ii) Backflow in the separated-flow region increases the effective velocity difference across the shear layer and hence tends to *increase* the shear stress and intensity.

(iii) Recirculation and re-entrainment of fluid deflected upstream at reattachment will tend to increase the shear stress and intensity; according to the measurements of Arie & Rouse and of Mueller & Robertson the shear stress in the reversed-flow region is certainly not negligible.

The angle at which the separated shear layer reattaches interacts with the entrainment rate and the properties of the separated-flow region. The interaction usually seems to be stable, although McEwan (1964) found that strongly oscillatory flow occurred if fluid was withdrawn through the back of a step. Addition or withdrawal of fluid due to spanwise flow is important if the ratio of 'bubble' thickness to span is not large; the present experiments, with a span of $30h$, showed severe three-dimensionality near the ends of the step.

### 5.2. *Bifurcation of the shear layer at reattachment*

At reattachment the shear layer splits. The fraction of the shear-layer mass flow that is deflected upstream depends on the initial boundary-layer thickness; if the latter is small at least half the shear layer may be deflected upstream to supply the entrainment. The following discussion refers to the case where a significant fraction is deflected.

Over most of the separated shear layer the dividing streamline lies near the line of maximum shear stress (Arie & Rouse 1956; Tani *et al.* 1961; Mueller & Robertson 1963), but near reattachment $\overline{uv}$ on the dividing streamline decreases rapidly, to a nominal value of zero at reattachment. The turbulence intensity on the dividing streamline also decreases, but less rapidly. The general behaviour of turbulence approaching a two-dimensional stagnation point has been treated theoretically by Hunt (private communication) and experimentally by Bearman (private communication). In the present case a large longitudinal rate of strain appears ($\partial U/\partial x$ reaches $0{\cdot}4\, U_1/h$ near the surface in the measurements of Tani *et al.*, compared with $\partial U/\partial y \doteqdot U_1/h$ in the main part of the shear layer) and the resulting increase in the longitudinal component of fluctuating vorticity would be expected to increase $\overline{v^2}$ and $\overline{w^2}$ near the surface while $\overline{u^2}$ will tend to decrease at the rate $-\overline{u^2}\,\partial U/\partial x$ (the extra production term in the equation for $D\overline{u^2}/Dt$). Opposing this will be the tendency for the normal component of velocity fluctuation to be *reduced* by the presence of the solid surface and to transfer energy to the $u$ and $w$ components by irrotational mechanisms; the outcome depends on the length scale of the fluctuations considered, so that severe distortion of spectrum shapes is likely.

The surface shear stress rises rapidly, by boundary-layer standards, after reattachment; $(h/\rho U_1^2)\,d\tau_w/dx$ is about 0·00006, 0·0002 and 0·0004 in the experiments of Tillman, Mueller & Robertson and the present authors respectively, but none of the measurements near reattachment are entirely reliable. For comparison, $(h/\rho U_1^2)\,\partial\tau/\partial y$ is of the order of 0·002 in the main part of the shear layer near reattachment so that lines of constant shear stress are still nearly parallel to the surface.

It remains to discuss the fate of the eddies entering the bifurcation region. Defining the larger eddies as bodies of fluid whose dimension in each direction is at least as large as the correlation integral length scale, we can see that the larger eddies in the separated shear layer, which are known (Bradshaw, Ferriss & Johnson 1964) to carry a large fraction of the shear stress, are themselves split. Whatever happens to the Reynolds stresses near reattachment, the *length* scales of the turbulence are likely to decrease significantly. Support for this statement comes from the behaviour of the dissipation length parameter $L$ in the optimized calculations (§4). In a plane mixing layer $L$, which is a typical scale of the energy-containing eddies, is about $0{\cdot}025x$, while the 'mixing length',

$$l \equiv (\tau/\rho)^{\frac{1}{2}}/(\partial U/\partial y),$$

is about $0{\cdot}016x$. Now in the present measurements at $x/h = 10$ we find that $l$ is about $0{\cdot}03x$ near the point of maximum shear stress (this is an isolated measurement; in the experiment of Tani *et al.* $l$ lay between $0{\cdot}018x$ and $0{\cdot}022x$ all the way from $x/h = 4$ to $x/h = 12$ which is again significantly higher than the value in a plane shear layer). Therefore in the *absence* of any changes in the interaction region we would expect $L$ to be, if anything, rather higher than in a plane mixing layer (about twice as large if $L/l$ is really the same). However the value of $L$ at $x/h = 10$ actually needed to optimize the calculations is $0{\cdot}012x$, or about *half* the value in a plane mixing layer. Whatever the above arguments may lack in rigour, the evidence for a sudden decrease of turbulence length scale as the shear layer splits seems fairly strong. The alternative is to suppose that the larger eddies are deflected alternately upstream and downstream rather than actually split, but one would expect this to lead to strong unsteadiness and McEwan (1964) found none.

Immediately downstream of reattachment the maximum shear stress, and the velocity gradient near the point of maximum shear stress, decrease rapidly (Tani *et al.*; Mueller & Robertson). The present measurements at $x/h = 10$ (figure 9) indicate that the shear correlation coefficient is 0·4–0·45, somewhat smaller than in a plane mixing layer but not small enough to suggest any strongly abnormal relation between the Reynolds stresses. The decrease in shear stress is attributable, according to most of the suggested constitutive equations for turbulence, to the supposed decrease in turbulence length scale. However the decrease in $\partial U/\partial y$ in the experiment of Tani *et al.* (1961) is quite spectacular and seems rather too large to be attributed to the action of the Reynolds stresses in

$$\left(U\frac{\partial}{\partial x}+V\frac{\partial}{\partial y}\right)\frac{\partial U}{\partial y}=-\frac{1}{\rho}\frac{\partial^2 p}{\partial x\,\partial y}-\frac{\partial^2\overline{uv}}{\partial y^2}-\frac{\partial^2\overline{u^2}}{\partial x\,\partial y},$$

and the pressure term is not entirely responsible for the discrepancy; however the evaluation of second derivatives from the measurements is very uncertain. What is certain is that the measured rate of decrease of $\partial U/\partial y$ with $x$ is much larger than an extrapolation of the trend in the mixing layer. One effect that *can* be attributed to pressure gradients is the rapid increase with $x$ of the velocity near the surface, which leads to a region of very small velocity gradient in the inner layer (e.g. see figure 8(*a*)) and initiates the dip below the logarithmic law that is maintained by the large gradient in turbulence length scale near the wall.

## 6. Conclusions

The model suggested by these observations is of a surprisingly complicated flow. The key to the behaviour of the relaxing boundary layer seems to be the rapid distortion of the shear layer near reattachment which depends on what fraction of the mass flow is deflected upstream to supply the entrainment. If the length of the separated-flow region is more than a few times the initial boundary-layer thickness this fraction is significant, and the perturbation is 'strong' or 'overwhelming' in the language of § 2. Not only do transverse pressure gradients and normal-stress gradients affect the flow, as is usual when the boundary-layer approximation fails, but the bifurcation of the shear layer produces large and immediate changes in the portion that continues to flow downstream. Since the dividing streamline is not too far from the line of maximum shear stress or turbulence intensity in the shear layer prior to reattachment, the large eddies that extend over most of the flow and produce much of the shear stress are roughly torn in two. The result is a rapid decrease in turbulent shear stress and there is strong indirect evidence that the turbulent length scales also decrease markedly. The practical conclusion is that the flow just downstream of reattachment bears very little resemblance to a plane mixing layer or any other sort of thin shear layer, even if the residual influence of the boundary layer at separation is small. A good deal of work will be needed to understand this unusual flow well enough to predict its behaviour in general.

After reattachment, the turbulent length scales, more especially the dissipation length parameter $L$, are roughly independent of $y$ except for a rapid decrease to a nominal value of zero at the surface. The effect of this rapid variation near the surface, compared to $L = Ky$ in an ordinary boundary layer, is to reduce the velocity gradient, and thus the velocity, in the inner layer below the value predicted by the logarithmic velocity profile formula. The internal boundary layer in which the mean velocity *does* follow the logarithmic law grows out from the surface at about the same rate as in the case of a change of surface roughness, but the 'law of the wall'–'law of the wake' formulation is inapplicable within a downstream distance of at least 30 times the shear-layer thickness at reattachment.

The rate at which the outer-layer structure (e.g. $L/\delta$) relaxes to the state of an ordinary boundary layer is very slow; the recovery of $L$ is satisfactorily represented in a fairly simple case by a 'time constant' of 25 times the shear-layer thickness at reattachment. Quantities like $c_f$ and the Clauser parameter $G$ revert to equilibrium constant-pressure values non-monotonically and much more slowly than has been suggested by previous experimenters.

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